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# Photon-assistant Fano resonance in serially coupled triple quantum dots

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## Abstract

Based on calculations of the electronic structure of serially coupled triple quantum dots (left, middle, right dots) (STQDs), we study the transport properties of the system driven by an ac electric field. It is found that the  $\Lambda$ -type energy level of STQDs has great impact on the transport properties. For an asymmetric (between left and right dots) configuration, there is a symmetric peak in the current-AC frequency curve due to resonant photon induced mixing between the left/right dot and the middle dot. In the symmetric (between left and right dots) configuration, a Fano asymmetric line shape appears in the current-frequency curve with the help of a 'trapping dark state'. Here the interesting coherent trapping phenomena, which usual appear in quantum optics, play an essential role in quantum electronic transport. We provide a clear physics picture for the Fano resonance and convenient ways to tune the Fano effects.

## 1. Introduction

The electronic transport in quantum dot structures is of great interest and has been the subject of active research in recent years. The strong spacial confinement leads to the discrete energy spectrum. These quantum dots (so called 'artificial atoms')/multiple coupled quantum dots (MCQDs, so called 'artificial molecules') are very important in understanding fundamental quantum phenomena. Many interesting phenomena which appear in real atoms and molecules can be well manifested in these nanostructures. Tunability provides many more opportunities for exploring richer physics which is hard to access in real atoms/molecules. It also leads to various applications. For instance, novel quantum logical gates and elementary qubits in quantum computers [1–3] can be realized based on these systems.

In these nanostructures, quantum coherence and interference is one of the most important issues and leads to a lot of interesting phenomena such as Aharonov–Bohm oscillations [4], Kondo effect [5, 6], coherent trapping [7, 8], Fano resonance [9–11] and so on. In recent years, triple quantum dots have attracted much attention due to their rich electric structure and physical phenomena. Some experimental groups have been able to fabricate triple quantum dots with high quality [12, 13], and many relevant theoretical studies have been

conducted on these systems [14–19]. In spite of many studies on the triple quantum dots, relatively little attention has been paid to the photon-assistant transport in triple quantum dots driven by a time-period electric field. Periodic driven triple quantum dots with the relevant three-level structure may have some interesting interference phenomena, such as coherent trapping, like those in quantum optics. In this paper we study the photon-assistant transport in serially coupled triple quantum dots (STQDs) paying special attention to the consequences of those interesting interference phenomena.

In previous theoretical work on MCQDs, the quantum levels, which depend on the structures of the system, are often assumed as parameters [20, 21], and the quantum properties of MCQDs cannot be presented quantitatively. Therefore, it is necessary to reveal the quantum behavior of MCQDs based on more realistic model. In our approach, we first design STQD with  $\Lambda$ -type three-level structure using a two-dimensional confining model in the effective mass frame. Based on the obtained level structure and with the help of Floquet theory [22], we study the transport properties of the STQDs. It is found that the  $\Lambda$ -type energy level has great impact on the transport properties: the photon-assistant tunneling leads to the symmetric Breit–Wigner [23] resonance when the system is asymmetric (between left and right quantum dots). When the system is in a symmetric (between left and right quantum dots) configuration, the formation of a 'trapping dark state' results

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in the interesting asymmetric Fano resonance under a resonant condition (photon energy equals the energy difference between the left/right quantum dot and the middle dot). Our studies on STQDs are not a straightforward extension of that for double quantum dot. There is no Fano effect in serial coupled double dots. The unique structure of STQDs leads to their interesting transport properties.

The organization of the paper is as follows. In section 2 we describe the model and calculate the electronic structure of the STQDs. In section 3 we present the photon-assistant transport properties of the system and discuss the results. The paper is summarized in section 4.

## 2. Model and electronic structure

We start our study by analyzing the electronic structure of STQDs. In experiments, STQDs are often fabricated from a heterostructure with a two-dimensional electron system. The confinement in the axial ( $Z$ ) direction of dots is far stronger than that in the  $X, Y$  directions. Only the lowest subband (due to confinement in  $Z$  direction) is relevant. Thus, in the theoretical modeling, the following two-dimensional potential is used

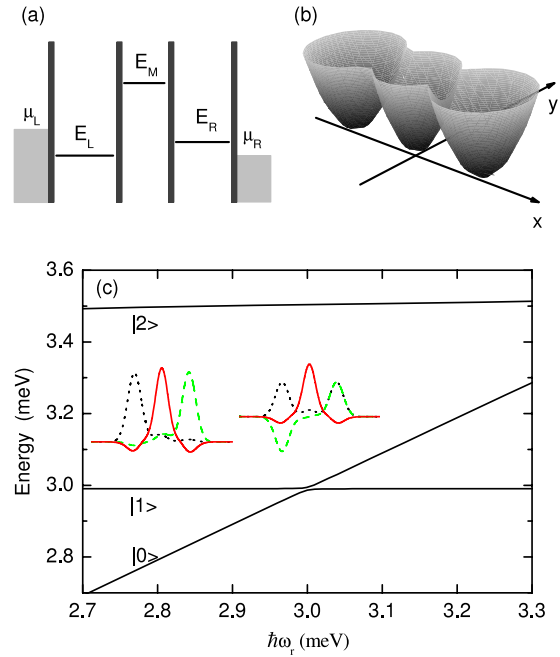
$$V(x, y) = 0.5m^* \min\{\omega_{lx}^2(x+d)^2 + \omega_{ly}^2y^2, \omega_{mx}^2x^2 + \omega_{my}^2y^2, \omega_{rx}^2(x-d)^2 + \omega_{ry}^2y^2\}, \quad (1)$$

where  $d$  is the interdot distance,  $m^*$  is the effective mass of the electron,  $\omega_{lx(y)}$ ,  $\omega_{mx(y)}$  and  $\omega_{rx(y)}$  are confining trap frequencies of the left, middle and right dots in the  $x(y)$  direction respectively. The model Hamiltonian of an electron in the coupled quantum dots can be written as

$$H = \frac{\mathbf{p}^2}{2m^*} + V(\mathbf{r}), \quad (2)$$

where  $\vec{\mathbf{r}} = (x, y)$ . In our calculation, the material parameter of GaAs QDs is used as  $m^* = 0.067m_e$  and the value of  $d$  is taken to resemble experimental systems. We use the eigenstates  $\varphi_i, \varphi_m, \varphi_r$  for each dot as the basis of the Hilbert space. Considering the nonorthogonality of the basis states, we obtain the eigenstates of equation (2) by solving the generalized eigenvalue of the system using diagonalization method<sup>1</sup>. Here, we investigate the levels of the STQDs by varying  $\hbar\omega_{rx} = \hbar\omega_{ry} = \hbar\omega_r$  with constant parameters of  $\hbar\omega_{lx} = \hbar\omega_{ly} = \hbar\omega_l = 3.00$  meV,  $\hbar\omega_{mx} = \hbar\omega_{my} = \hbar\omega_m = 3.52$  meV, and  $d = 77.70$  nm. For such parameters, the tunneling energy between the left and middle dot is about 51  $\mu$ eV. Experimentally, the levels of each dot are tuned by changing the voltage of the corresponding electrode gate. Such change of voltage corresponds to the variation of confining strength of the dot in our model. In figure 1(c), the lowest three eigenenergy levels of the STQD are plotted as functions of  $\hbar\omega_r$ . As the value of  $\hbar\omega_r$  increases, the level of the right dot increases. When  $\hbar\omega_r = \hbar\omega_l = 3.00$  meV, where the levels of left and right dots are equal, the states  $|0\rangle$  and  $|1\rangle$  becomes a pair of delocalized bonding and antibonding states,

<sup>1</sup> See the library of Fortran or Matlab for the generalized eigenvalue problem  $\mathbf{AX} = \lambda \mathbf{BX}$ .



**Figure 1.** (a) Expected  $\Lambda$ -type three-level scheme for STQDs. (b) Confining potential model of STQDs. (c) The lowest three eigenenergy levels as functions of  $\hbar\omega_r$  for STQDs of  $\hbar\omega_l = 3.00$  meV,  $\hbar\omega_m = 3.52$  meV, and  $d = 77.70$  nm. Inset: eigenwavefunctions at  $\hbar\omega_r = 3.00$  meV and 2.70 meV. The solid line is for state  $|2\rangle$ . The dashed line is for state  $|1\rangle$ . The dotted line is for state  $|0\rangle$ .

(This figure is in colour only in the electronic version)

while the other state  $|2\rangle$  is localized in the middle quantum dot. The corresponding eigenwavefunctions are shown in the inset. When  $\hbar\omega_r$  is away from 3.00 meV, the three levels are nearly energies of the ground states of the left, middle and right dots ( $|L\rangle, |M\rangle, |R\rangle$ ) respectively, and the eigenstates are all localized in each quantum dot, as shown from their wavefunctions in the inset. The continuous manifolds on the two sides of the figure 1(a) correspond to electronic states with chemical potential  $\mu_L$  and  $\mu_R$  and the two side dots are coupled to the leads with the dots-lead hopping rate  $\Gamma_L$  and  $\Gamma_R$ .

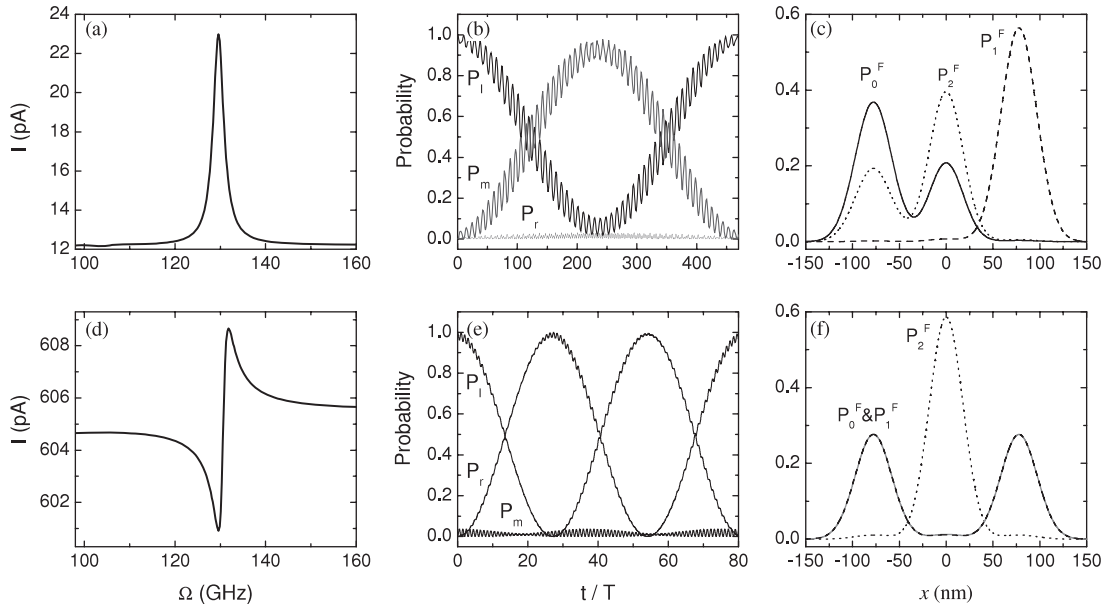
## 3. Photon-assistant transport properties

Based on the results of electronic structure, we study the transport properties through the system with a  $\Lambda$ -type three-level structure under the action of an ac driving field with a frequency  $\Omega$ . The transport properties of this time-dependent system can be studied with the help of Floquet theorem. It can be shown that the average current of this system can be written as [22]

$$I = \frac{e}{h} \sum_{k=-\infty}^{+\infty} \int d\varepsilon \{T_{LR}^{(k)}(\varepsilon) f_R(\varepsilon) - T_{RL}^{(k)}(\varepsilon) f_L(\varepsilon)\}, \quad (3)$$

where

$$T_{LR}^{(k)}(\varepsilon) = \Gamma_L(\varepsilon + k\hbar\Omega)\Gamma_R(\varepsilon)|\langle L|G^{(k)}(\varepsilon)|R\rangle|^2 \quad (4)$$



**Figure 2.** (a) Average current  $I$  as a function of the driving frequency  $\Omega$ . (b) Time-dependent occupation probabilities for an electron in the left, middle and right dots. (c) Time average of electron probability distributions of the Floquet states at  $\Omega = 129.72$  GHz where  $\hbar\omega_r = 3.10$  meV,  $k_B T = 0$ . (d), (e), (f) are the same as (a), (b), (c) respectively except that  $\Omega = 130.68$  GHz and  $\hbar\omega_r = 3.00$  meV.

$$T_{RL}^{(k)}(\varepsilon) = \Gamma_R(\varepsilon + k\hbar\Omega)\Gamma_L(\varepsilon)|\langle R|G^{(k)}(\varepsilon)|L\rangle|^2 \quad (5)$$

denote the transmission probabilities for electrons with initial energy  $\varepsilon$  and final energy  $\varepsilon + k\hbar\Omega$  from the right lead and from the left lead, respectively.  $f_\ell(\varepsilon) = (1 + \exp[(\varepsilon - \mu_\ell)/k_B T])^{-1}$  denotes the Fermi function. According to Floquet theorem for a time-periodic Hamiltonian, there exists a complete set of solutions for the Schrodinger equation, which have the form  $e^{-i(\varepsilon_\alpha + \gamma_\alpha)t}|u_\alpha(t)\rangle$ , where Floquet states  $|u_\alpha(t)\rangle$  are time-periodic functions, and  $\varepsilon_\alpha, \gamma_\alpha$  ( $\alpha = 0, 1, 2$ ) are the real and imaginary parts of quasi-energies respectively. The Fourier coefficients of the retarded Green function can be written as

$$G^{(k)}(\varepsilon) = \sum_{\alpha, k'} \frac{|u_{\alpha, k'+k}\rangle \langle u_{\alpha, k'}^+|}{\varepsilon - (\varepsilon_\alpha + k'\hbar\Omega - i\gamma_\alpha)}, \quad (6)$$

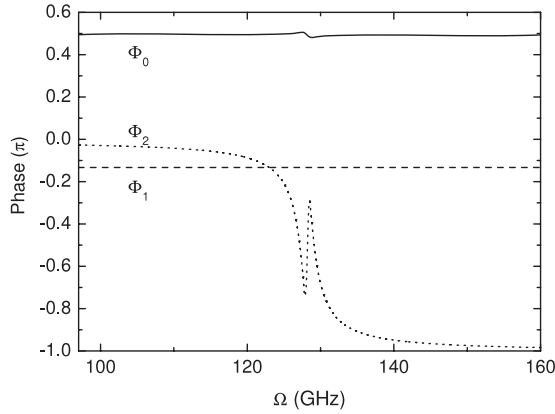
where  $|u_{\alpha, k}\rangle$  are the Fourier coefficients of the Floquet state  $|u_\alpha(t)\rangle$ . In our numerical calculations, we set the energy independent dots-lead hopping rate  $\Gamma_L = \Gamma_R = 9 \mu\text{eV}$ , and assume that  $k_B T = 0$ , then the Fermi functions in the expression for the average current (3) become step functions. The ac field magnitude  $A = 1.54 \text{ V cm}^{-1}$ . Beside the AC field, we have also applied a small DC voltage, i.e.  $\mu_L - \mu_R = 300 \mu\text{V}$ , to ensure a nonvanishing time average current.

We first study the transport properties of an asymmetric system by applying different confining potentials to the left and right dots. The average current  $I$  as a function of the driving frequency  $\Omega$  with  $\hbar\omega_r = 3.10$  meV is presented in figure 2(a). We find the current curve has a symmetric Breit–Wigner line shape around  $\Omega = 129.72$  GHz, suggesting that there is a resonance for electrons in the system in this case ( $\hbar\Omega = E_2 - E_0$ ). The time evolution of the probabilities for an electron in left, middle, right dot are shown in figure 2(b). Here we have performed our calculation in a closed system (i.e., without

interaction with the leads) and have used the initial condition  $P_L = 1$  and the resonant condition  $\Omega = 129.72$  GHz. This calculation shows how the interactions between the dots and with the AC field lead to the mixing of states  $|L\rangle, |M\rangle$  and  $|R\rangle$ . In the case of small  $\Gamma$ , the interaction with leads determines the width of states  $|L\rangle$ , and  $|R\rangle$ , and has little effect on the mixing of states  $|L\rangle, |M\rangle$  and  $|R\rangle$ . Figure 2(c) shows the time average of the probability distribution of Floquet states,  $P_i^F = \frac{1}{T} \int_0^T |u_i(t)|^2 dt$ ,  $i = 0, 1, 2$ . It is clear that there is a photon-assistant mixing between left and middle dot. This photon-assistant charge transfer leads to the occurrence of the current resonance phenomenon.

Then we consider the case that the system is symmetric by applying the same confining potential to the two side dots. The average current  $I$  as a function of the driving frequency  $\Omega$  with  $\hbar\omega_r = 3.00$  meV is displayed in figure 2(d). We find that the current curve has an asymmetric Fano line shape around  $\Omega = 130.68$  GHz and the current amplitude is much larger than that in the asymmetric case. In order to understand the intriguing phenomenon, we did a similar calculation of the time evolution of the occupation probabilities for an electron in the left, middle and right dot and the time average of probability distribution of Floquet states. As shown in figures 2(e) and (f), we find that two delocalized Floquet states (which are related to bonding and antibonding states) are formed; another Floquet state is still localized in the middle dot in this case, indicating that the middle dot mediates the super-exchange interaction between the left and right dots. The states  $|0\rangle, |1\rangle$  behave like a ‘trapping dark state’ in an atomic system. Though state  $|2\rangle$  has a very small wavefunction amplitude in the left and right dots, it plays an important role in the occurrence of the Fano-type resonance, which is presented in the following discussion.

As seen from equation (6), three Floquet states form three conducting channels. Due to the interaction with leads, the



**Figure 3.** Phases as functions of the driving frequency  $\Omega$ .

three states may have different widths. In the symmetric situation, the bonding and antibonding Floquet states  $|u_0\rangle$ ,  $|u_1\rangle$  have large wavefunction amplitudes in the left and right dots, and thus are strongly coupled to the leads. They have a large width and can be viewed as continuous channels. The localized Floquet state  $|u_2\rangle$  has small wavefunction amplitude in the left and right dots and is weakly coupled to the leads. It has a very narrow width and can be viewed as resonant discrete channels. The interference between strongly coupled channels and weakly coupled channel leads to the observed Fano resonance [24]. While in the asymmetric situation, the Floquet state  $|u_2\rangle$  is also effectively coupled to the leads via a photon-assistant mixing between the left and middle dot. Thus all three states form strongly coupled channels, so no Fano effect is observed. Here we discuss in detail the interference between different channels for the symmetric system. From equation (6), one can see that the average current  $I$  is mainly decided by  $|\langle R|G^{(k)}(\varepsilon)|L\rangle|^2$ . The interference effects can be seen from following calculation (here we show the term with  $k = 0$ ,  $\varepsilon = 150 \mu\text{V}$  as an example)

$$\langle R|G^{(0)}(\varepsilon)|L\rangle = \sum_{\alpha,k'} \frac{\langle R|u_{\alpha,k'}\rangle \langle u_{\alpha,k'}^+|L\rangle}{\varepsilon - (\varepsilon_{\alpha} + k'\hbar\Omega - i\gamma_{\alpha})} = \sum_{\alpha} A_{\alpha} e^{i\phi_{\alpha}}. \quad (7)$$

In figure 3, we show the phases  $\phi_{\alpha}(\alpha=0,1,2)$  as the functions of the driving frequency  $\Omega$ . Here, the phases  $\phi_0$  and  $\phi_1$  are phases of two strongly coupled states  $|u_0\rangle$  and  $|u_1\rangle$ , and  $\phi_2$  is the phase of the weakly coupled state  $|u_2\rangle$ . From figure 3, we find that  $\phi_2$  has a phase shift that varies from 0 to  $\pi$  as the field frequency  $\Omega$  is moved through the resonance frequency, which provides a resonant channel for electrons to transport the system.  $\phi_0$ ,  $\phi_1$  are nearly constant and can be assumed to be independent of the driving frequency, which can be considered as background or nonresonant channels. Therefore, the interference between the strongly coupled channels and weakly coupled channel results in the Fano-type resonance. Finally, we would like to point out that the above picture also explains why there is no Fano resonance in a serially coupled double dot. As a matter of fact, in that situation, there are only strongly coupled bonding and antibonding states. Therefore, there is no apparent Fano effect.

## 4. Conclusion

In summary, we have studied the transport properties of STQDs under the action of an ac electric field. Using a two-dimensional confining potential model in the effective mass frame, the  $\Lambda$ -type three-level structure is obtained through solving the generalized eigenvalue of the system. Based on the level structure and Floquet theory, we investigate the dependence of the ac current on the electronic structure of the system. It is found that when the system is asymmetric, the symmetric Breit–Wigner resonance appears due to phonon-assistant tunneling. When the system is symmetric, the interesting asymmetric Fano resonance occurs under the resonant condition. In this case, quantum interference results in the formation of ‘trapping dark states’. These ‘trapping dark states’ are the delocalized bonding and antibonding states which serve as the continuous channels. Here the middle dot plays a dual role: (1) it mediates the super-exchange interaction between the left and the right dots; (2) the localized state within it serves as the resonant discrete channel. Our work presents the unique quantum interference features of the multiple quantum dots system and is useful for the design of novel nanodevices.

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